Solving MINLPs with a MIP Solver

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Motivation

NLP applications:

- **Revenue Management**
- Gas networks
- Power systems
- Physics

An example from RM:

$$max_{\beta \in \mathbb{R}^n} \sum_{j=1}^{n} K_j \beta_j - \sum_{t=1}^{T} m_t log \left(\sum_{i \in S_t} e^{\beta_i} \right)$$

$$s.t. \sum_{i=1}^{n} e^{\beta_i} = \frac{s}{1-s}$$

Motivation

What to do?

- Hope that your functions are handled by a non-linear solver.
- Solver choices:
  - ANTIGONE
  - Alpine
  - BARON
  - Bonmin
  - Couenne
  - Knitro...
- Hope that you have the money to buy the solver.

Motivation

From Baron user manual:
"**2.1 Allowable nonlinear functions** In addition to multiplication and division, BARON can handle nonlinear functions that involve e^x, $ln(x)$, x^α for real α, and β^x for real β. AIMMS/BARON, AMPL/BARON, andGAMS/BARON automatically handle $|x|$ and xy, where x and$y$ are variables; otherwise, suitable transformations discussed below can be used. There is currently no support for other functions,including the trigonometric functions $sin(x)$, $cos(x)$, etc.

Contributions

Informally, we give you a way to solve most NLPs with CPLEX.
Formally, given a function $f : \mathbb{R} \to \mathbb{R}$ with the following properties:

- continuous,
- differentiable,
- has bounded domain,
- domain can be partitioned into regions such that f is either strictly convex or strictly concave in each region,

We provide:

- An infinite sequence of MIP relaxations that converge to $f(x)$.
- An infinite sequence of LP relaxations that converge to the convex closure of $f(x)$.
- Both converge fast (quadratic).

Caveats

- Cannot handle multi-linear terms like $z = xy$ or $w = xyz$ (but relaxations exist for these).
- Relaxations are weak for well-known convex high dimensional surfaces (as of yesterday). For example, Lorentz cones: $\sqrt{x^2 + y^2 + z^2} \leq t$.

$x^4 - x^3$ MIP convergence

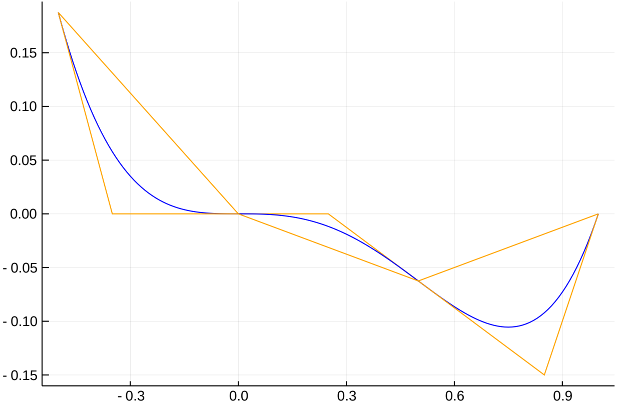


Figure: Coarse partition

$x^4 - x^3$ MIP convergence

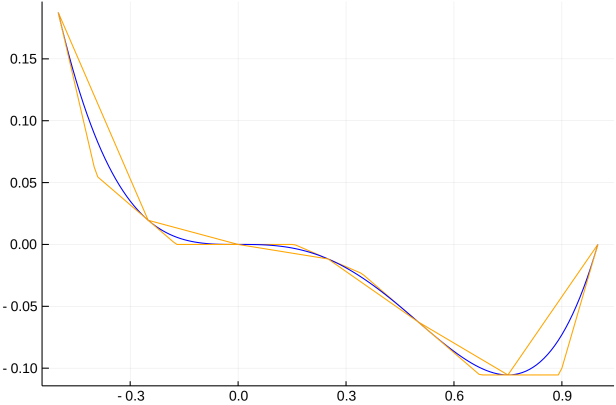


Figure: Medium partition

$x^4 - x^3$ MIP convergence

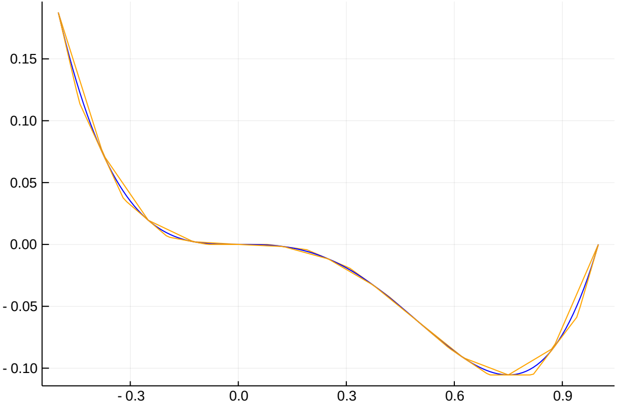
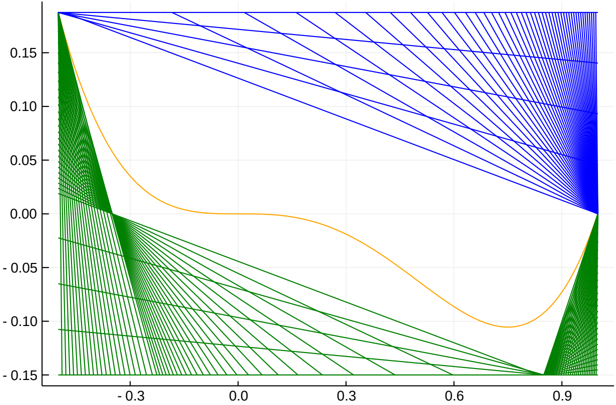


Figure: Fine partition

$x^4 - x^3$ LP convergence



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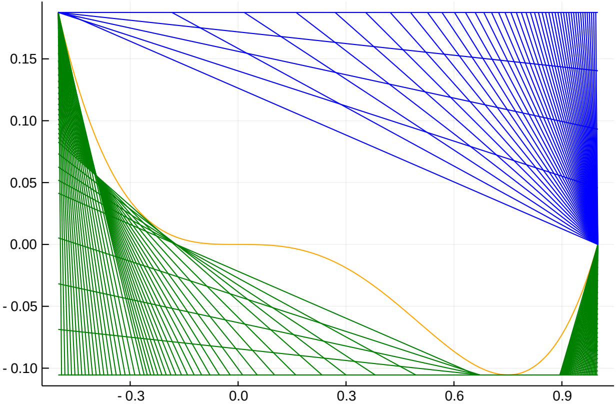

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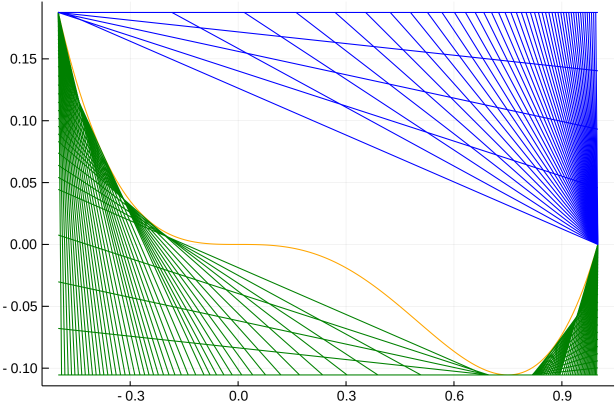

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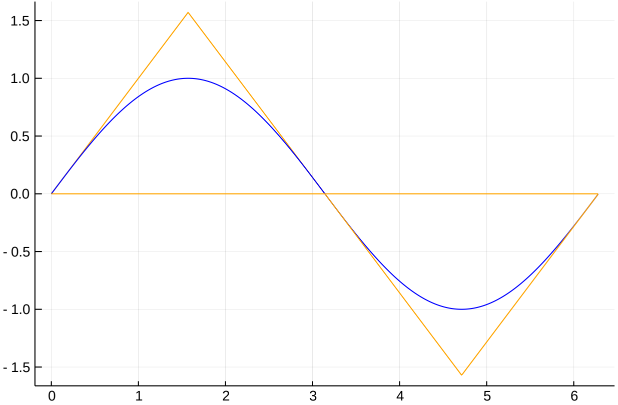
$sin(x)$ MIP convergence



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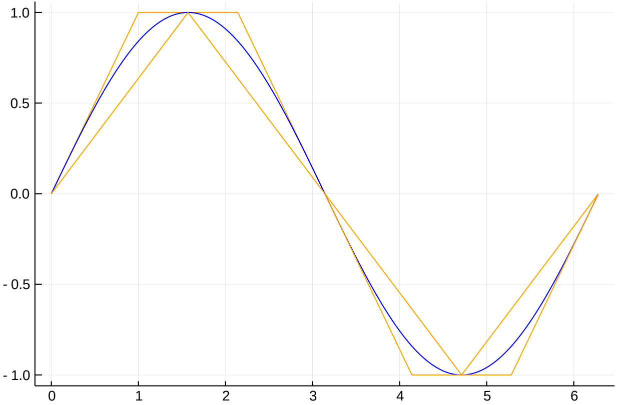

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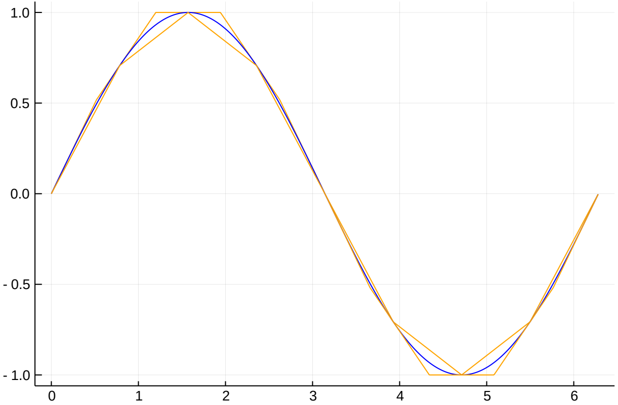

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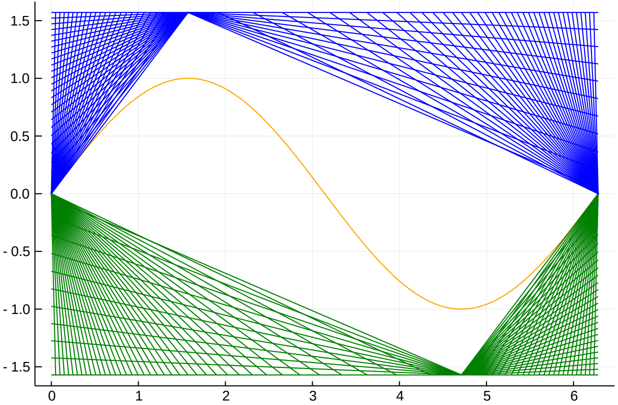

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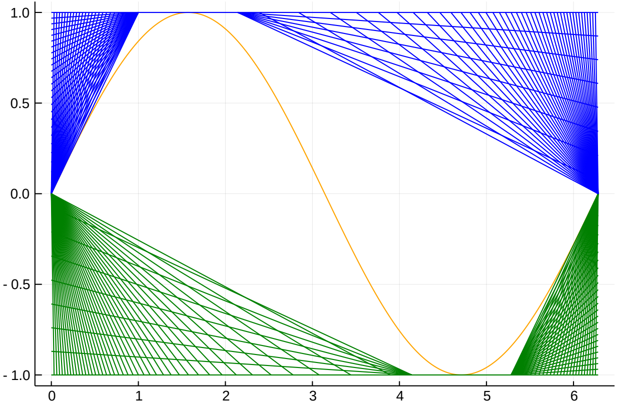

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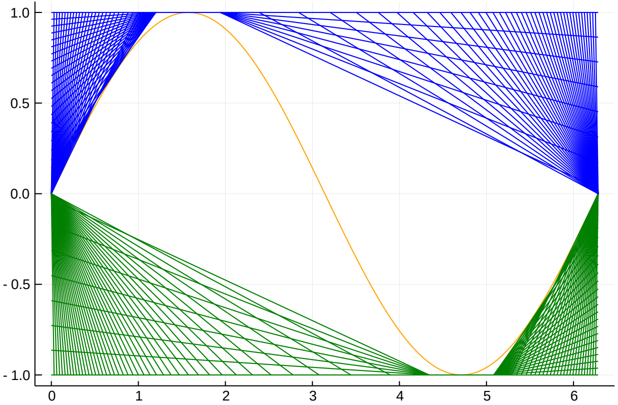

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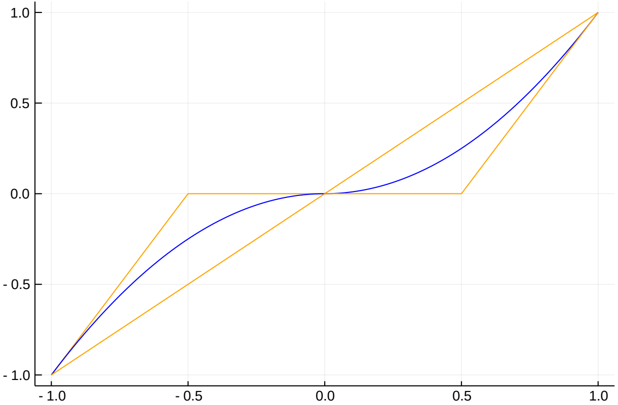
$x|x|$ MIP convergence



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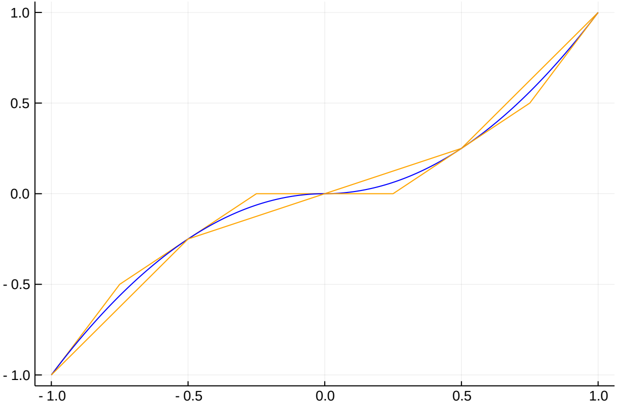
$x|x|$ *MIP* convergence



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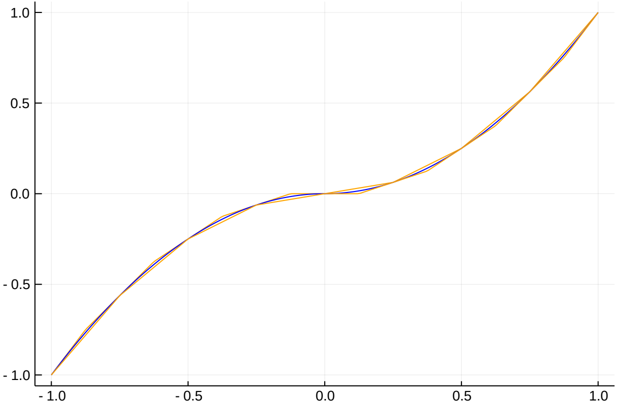
$x|x|$ MIP convergence



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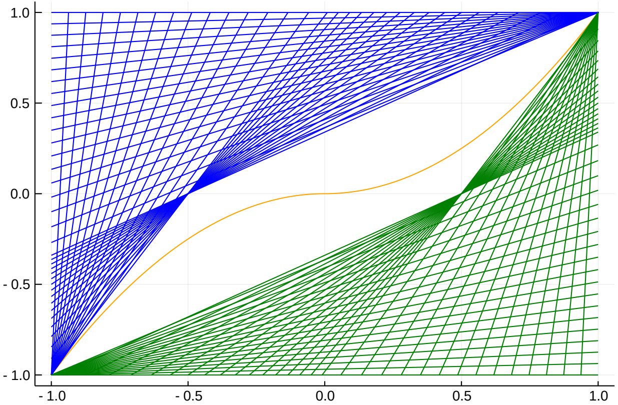
$x|x|$ LP convergence



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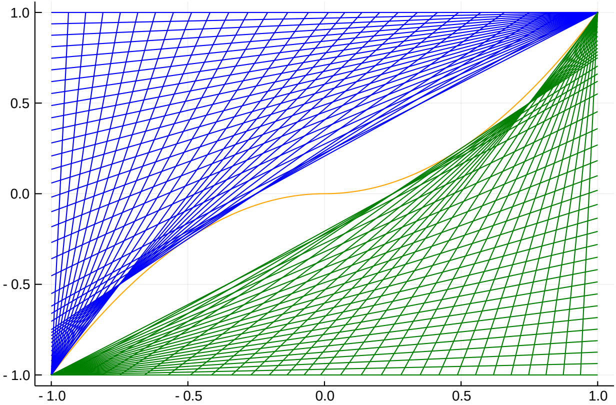

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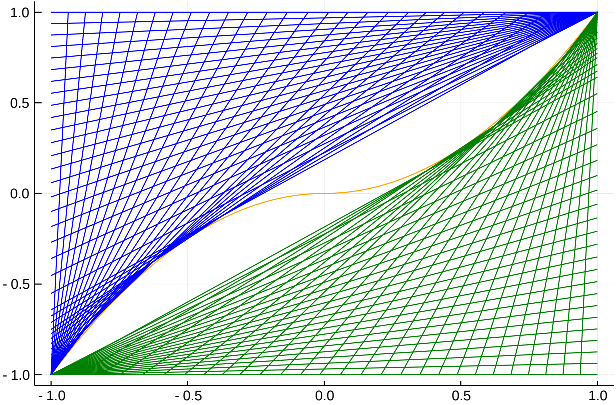

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