# Managing Airline Delay Uncertainty by Adjusting Connection Slacks

## A Stochastic Programming Approach

Sujeevraja Sanjeevi[1] Saravanan Venkatachalam[2]

[1]Team Lead Operations Research
Sabre, TX, USA

[2]Assistant Professor
Wayne State University, MI, USA

INFORMS, October 2019

# Contents

# Table of Contents

# Delays

- Domestic flight delay impact to the U.S. economy is $32.9 billion, Ann Brody Guy 2010.
- $988,043$ (19.95%) flights of reporting carriers with arrival delays in 2019, Bureau of Transportation Statistics 2019.
- In Europe, en route delays doubled in 2018 compared with 2017, to more than 19 million minutes, IATA 2019.

# Managing Delays

*Research Motivation*

- Re-timing flights is expensive on day-of-ops.
- Account for uncertain delays when planning.
- Delay-friendly schedule.

*Classifying flight delays*

- Primary delay: independent of routing.
- Propagated delay: caused by up-stream flights.

# Literature

- Lan, Clarke, and Barnhart 2006
  - Minimize $\mathbb{E}$(*propagated delays*) on selected routes.
  - Discrete delay copies.
- Dunbar, Froyland, and Wu 2014
  - Minimize $\mathbb{E}$(*propagated delays*) on selected routes.
  - Iterate between delay propagation, routing and crew planning.
- Yan and Kung 2016
  - Robust optimization.
  - Minimize maximal total propagated delay.

# Our Contributions

- Balance cheap rescheduling today and costly uncertain routing/rescheduling tomorrow.
- Two-stage stochastic programming model.
- Solution framework with L-shaped method and column generation.
- Extensive computational study.

# Table of Contents

# Two Stage Model (TSM)

$$
\begin{array}{lll}
\text{Min} & c^T x + \mathbb{E}_\Omega[\phi(x,\omega)] & \\
\text{s.t} & x_i \leq s_{ij} + x_j, & (i,j) \in A^{orig} \\
& \sum_{f \in F} x_f \leq b, & \\
& x_f \in \mathbb{Z} \cap [0, l]. & f \in F
\end{array}
$$

- $x_f$: reschedule time of flight $f$
- $s_{ij} = dep_j - (arr_i + turn_{ij})$
- $\Omega$: delay scenarios
- $\mathbb{E}_\Omega[\phi(x,\omega)] = \sum_{\omega \in \Omega} p^\omega \phi(x,\omega)$

# Recourse Model

$$
\begin{aligned}
\phi(x,\omega) = \text{Min} \quad & e^T z^\omega \\
\text{s.t.} \quad & \sum_{r \in RT_i^\omega} y_r^\omega = 1, && i \in T, \\
& \sum_{r \in RF_f^\omega} y_r^\omega = 1, && f \in F, \\
& \sum_{r \in RF_f^\omega} pd_{rf}^\omega y_r^\omega - x_f \leq z_f^\omega && f \in F, \\
& z_f^\omega \geq 0,\ f \in F, y_r^\omega \in \{0,1\},\ r \in R^\omega
\end{aligned}
$$

- $z_f^\omega$: excess delay of flight $f$.
- $y_r^\omega$: 1 if route $r$ is selected, 0 otherwise.
- $pd_{rj}^\omega = (pd_{ri}^\omega + d_i^\omega - s_{ij})_+$.

# Recourse Model with Expected Excess

$$
\begin{aligned}
\phi(x,\omega) = \text{Min} \quad & e^T z^\omega + \lambda v^\omega \\
\text{s.t.} \quad & \sum_{r \in RT_i^\omega} y_r^\omega = 1, && i \in T, \\
& \sum_{r \in RF_f^\omega} y_r^\omega = 1, && f \in F, \\
& \sum_{r \in RF_f^\omega} pd_{rf}^\omega y_r^\omega - x_f \le z_f^\omega && f \in F, \\
& c^T x + e^T z^\omega - \alpha \le v^\omega, \\
& z_f^\omega \ge 0,\ f \in F, y_r^\omega \in \{0,1\},\ r \in R^\omega, v^\omega \ge 0
\end{aligned}
$$

- $v^\omega$: excess cost.
- $\lambda$: risk aversion, $\alpha$: risk target.
- minimizes $c^T x + \mathbb{E}_\Omega[(\phi(x,\omega) - \alpha)_+]$

# Mean Delay Model (MDM)

$$
\begin{array}{lll}
\text{Minimize} & c^T x + e^T z^{\bar{\omega}} & \\
\text{s.t.} & x_i \le s_{ij} + x_j, & (i,j) \in A^{orig}, \\
 & \displaystyle\sum_{f \in F} x_f \le b, & \\
 & pd_f^{\bar{\omega}} - x_f \le z_f^{\bar{\omega}}, & f \in F, \\
 & z_f^{\bar{\omega}} \ge 0, x_f \in \mathbb{Z} \cap [0, l], & f \in F.
\end{array}
$$

- $\bar{\omega}$: scenario with average delays.
- $pd_f^{\bar{\omega}}$: propagated delay on original route.

# Table of Contents

# Decomposition Framework

**Algorithm** *L*-shaped algorithm

Solve master problem (MP) to obtain initial solution $x^0$
Set $UB \leftarrow \infty$, $LB \leftarrow -\infty$, $k \leftarrow 0$
**while** UB - LB $> \epsilon$ **do**
  Find $\phi(x^k, \omega)$ for $\omega \in \Omega$ (in parallel)
  $UB \leftarrow min(UB, \sum_{\omega \in \Omega} p_\omega \phi(x^k, \omega))$
  Add Benders cut(s) and solve MP (single/multi cut)
  Update $z^* \leftarrow z^k$ and $x^* \leftarrow x^k$
  LB $\leftarrow max(LB, MP\ objective)$, $k \leftarrow k + 1$
**end while**

- $\phi(x, \omega)$: solved with column generation.

# Table of Contents

# Data

Table: Flight Networks

| *Name* | *Flights* | *Aircraft* | *Paths* |
|---|---|---|---|
| s1 | 210 | 41 | 48,674 |
| s2 | 248 | 67 | 20,908 |
| s3 | 112 | 17 | 39,242 |
| s4 | 110 | 17 | 56,175 |
| s5 | 80 | 13 | 190,540 |

# Parameters (data)

- Delayed flights: hub, rush (first quarter of day)
- Distributions: log-normal, exponential, truncated normal
- Means: 15, 30, 45, 60 minutes
- Budgets: 0.25, 0.5, 0.75, 1, 2 times average primary delay
- Flight reschedule limit: 30 minutes

# Parameters (algorithm)

- Benders iterations: 30
- Parallel solvers: 30
- Number of recourse scenarios: 30
- Number of simulation scenarios: 100
- Column selection: first 10, best 10, all, full enumeration
- $L$-shaped cuts: single cut, multi cut

# Computational Setup

- Language: Java
- Solver: CPLEX 12.9
- System:
  - Intel(R) Xeon(R) CPU E5-2640
  - 80 GB RAM
  - 8 cores, 16 logical cores
- Parallelism: Akka actor framework (`https://akka.io`)

# Quality and Performance

| *Delays* | *Name* | *Time* | *Gap (%)* | *Opt gap (%)* | *Cuts* | *Iter* |
|---|---|---|---|---|---|---|
| Hub | s1 | 78.42 | 0.35 | 3.42 | 886 | 30 |
| | s2 | 53.94 | 2 | 3.87 | 900 | 30 |
| | s3 | 15.94 | 0 | 0 | 93 | 6 |
| | s4 | 14.04 | 0.05 | 7.61 | 304 | 15 |
| | s5 | 73.16 | 0 | 6.18 | 352 | 16 |
| Rush | s1 | 90.64 | 0.09 | 7.52 | 861 | 30 |
| | s2 | 71.07 | 0.5 | 7.94 | 888 | 30 |
| | s3 | 11.73 | 0.03 | 8.75 | 79 | 4 |
| | s4 | 6.37 | 0 | 0.41 | 115 | 6 |
| | s5 | 47.92 | 0 | 0.09 | 188 | 8 |

# Changing Budgets

Table: Propagated delay improvement for budget fraction 0.25

| *Name* | *Original* | *MDM* | *RR (%)* | *TSM* | *RR (%)* |
|---|---|---|---|---|---|
| s1 | 845 | 628.06 | 25.67 | 562.57 | **33.42** |
| s2 | 850.82 | 611.65 | 28.11 | 520.17 | **38.86** |
| s3 | 50.24 | 26.88 | 46.5 | 15.68 | **68.79** |
| s4 | 219.37 | 145.93 | 33.48 | 135.86 | **38.07** |
| s5 | 254.29 | 215.02 | 15.44 | 160.18 | **37.01** |

# Changing Budgets

Table: Propagated delay improvement for budget fraction 0.5

| *Name* | *Original* | *MDM* | *RR (%)* | *TSM* | *RR (%)* |
|---|---|---|---|---|---|
| s1 | 836.37 | 474.79 | 43.23 | 406.51 | **51.4** |
| s2 | 844.62 | 416.29 | 50.71 | 363.95 | **56.91** |
| s3 | 42.45 | 19.89 | 53.14 | 8.6 | **79.74** |
| s4 | 232.55 | 150.1 | 35.45 | 117.32 | **49.55** |
| s5 | 250.1 | 123.74 | 50.52 | 115.61 | **53.77** |

# Changing Budgets

Table: Propagated delay improvement for budget fraction 0.75

| *Name* | *Original* | *MDM* | *RR (%)* | *TSM* | *RR (%)* |
|---|---|---|---|---|---|
| s1 | 861.65 | 373.57 | 56.64 | 365.71 | **57.56** |
| s2 | 868.94 | 345.26 | 60.27 | 303.68 | **65.05** |
| s3 | 46.81 | 25.88 | 44.71 | 11.76 | **74.88** |
| s4 | 218.15 | 132.55 | 39.24 | 87.93 | **59.69** |
| s5 | 242.06 | 116.37 | 51.93 | 102.03 | **57.85** |

# Changing Budgets

Table: Propagated delay improvement for budget fraction 1

| *Name* | *Original* | *MDM* | *RR (%)* | *TSM* | *RR (%)* |
|---|---|---|---|---|---|
| s1 | 832.36 | 349.93 | 57.96 | 272.63 | **67.25** |
| s2 | 829.33 | 316.21 | 61.87 | 209.45 | **74.74** |
| s3 | 49.48 | 29.62 | 40.14 | 19.71 | **60.17** |
| s4 | 233.37 | 155.23 | 33.48 | 106.54 | **54.35** |
| s5 | 246.86 | 123.38 | 50.02 | 89.9 | **63.58** |

# Changing Budgets

Table: Propagated delay improvement for budget fraction 2

| *Name* | *Original* | *MDM* | *RR (%)* | *TSM* | *RR (%)* |
|---|---|---|---|---|---|
| s1 | 849.18 | 351.68 | 58.59 | 238.15 | **71.96** |
| s2 | 851.63 | 344.38 | 59.56 | 222.88 | **73.83** |
| s3 | 49.12 | 28.81 | 41.35 | 16.94 | **65.51** |
| s4 | 222.53 | 144.08 | 35.25 | 95.3 | **57.17** |
| s5 | 243.47 | 116.92 | 51.98 | 79.63 | **67.29** |

# Changing Distributions

Table: Propagated delay improvement for *Exponential*(30)

| *Name* | *Original* | *MDM* | *RR (%)* | *TSM* | *RR (%)* |
|---|---|---|---|---|---|
| s1 | 2050.08 | 1562.11 | 23.8 | 1230.08 | **40** |
| s2 | 1993.59 | 1336.85 | 32.94 | 1107.84 | **44.43** |
| s3 | 141.43 | 87.52 | 38.12 | 55.89 | **60.48** |
| s4 | 701.25 | 434.87 | 37.99 | 391.68 | **44.15** |
| s5 | 599.99 | 411.45 | 31.42 | 330.68 | **44.89** |

# Changing Distributions

Table: Propagated delay improvement for $LogNormal(30, 15)$

| *Name* | *Original* | *MDM* | *RR (%)* | *TSM* | *RR (%)* |
|---|---|---|---|---|---|
| s1 | 1966.24 | 1233.31 | 37.28 | 867.31 | **55.89** |
| s2 | 1849.07 | 999.45 | 45.95 | 663.77 | **64.1** |
| s3 | 116.12 | 46.47 | 59.98 | 24.7 | **78.73** |
| s4 | 575.49 | 223.43 | 61.18 | 203.98 | **64.56** |
| s5 | 557.96 | 310.85 | 44.29 | 209.47 | **62.46** |

# Changing Distributions

Table: Propagated delay improvement for $TruncNormal(30, 15)$

| *Name* | *Original* | *MDM* | *RR (%)* | *TSM* | *RR (%)* |
|---|---|---|---|---|---|
| s1 | 2008.96 | 1204.15 | 40.06 | 903.91 | **55.01** |
| s2 | 1919.41 | 900.75 | 53.07 | 693.99 | **63.84** |
| s3 | 115.87 | 39.72 | 65.72 | 18.16 | **84.33** |
| s4 | 615.21 | 248.11 | 59.67 | 207.77 | **66.23** |
| s5 | 580.18 | 378.54 | 34.75 | 210.44 | **63.73** |

# Changing Means

Table: Propagated delay improvement for *Exponential*(15)

| *Name* | *Original* | *MDM* | *RR (%)* | *TSM* | *RR (%)* |
|---|---|---|---|---|---|
| s1 | 860.29 | 521.5 | 39.38 | 472.28 | **45.1** |
| s2 | 853.49 | 453.26 | 46.89 | 395.58 | **53.65** |
| s3 | 42.41 | 23.41 | 44.8 | 9.34 | **77.98** |
| s4 | 235.08 | 155.06 | 34.04 | 122.52 | **47.88** |
| s5 | 252.87 | 149.5 | 40.88 | 122.54 | **51.54** |

# Changing Means

Table: Propagated delay improvement for *Exponential*(45)

| *Name* | *Original* | *MDM* | *RR (%)* | *TSM* | *RR (%)* |
|---|---|---|---|---|---|
| s1 | 3504.48 | 2554.76 | 27.1 | 2286.38 | **34.76** |
| s2 | 3079.29 | 1930.02 | 37.32 | 1818.79 | **40.93** |
| s3 | 267.3 | 166.12 | 37.85 | 142.39 | **46.73** |
| s4 | 1199.09 | 762.59 | 36.4 | 703.14 | **41.36** |
| s5 | 1042.92 | 723.06 | 30.67 | 653.13 | **37.37** |

# Changing Means

Table: Propagated delay improvement for *Exponential*(60)

| *Name* | *Original* | *MDM* | *RR (%)* | *TSM* | *RR (%)* |
|---|---|---|---|---|---|
| s1 | 5247.03 | 3922.66 | 25.24 | 3715.71 | **29.18** |
| s2 | 4674.16 | 3045.3 | 34.85 | 2938.05 | **37.14** |
| s3 | 412.07 | 280.51 | 31.93 | 257.87 | **37.42** |
| s4 | 1825.04 | 1322.64 | 27.53 | 1168.95 | **35.95** |
| s5 | 1437.66 | 1138.58 | 20.8 | 958.5 | **33.33** |

# Column Selection

Table: Time comparison for column generation strategies

| *Name* | *Enumeration* | *All paths* | *Best paths* | *First paths* |
|---|---|---|---|---|
| s1 | 958.58 | 112.33 | 75.61 | 77.45 |
| s2 | 161.19 | 63.45 | 47.46 | 49.87 |
| s3 | 170.61 | 19.64 | 9.87 | 9.49 |
| s4 | 417.46 | 28.32 | 15.20 | 14.28 |
| s5 | 3086.92 | 121.61 | 65.81 | 69.34 |

# Parallelism

Table: Run-time comparison for parallel sub-problems

| | *Number of parallel solvers* | | | |
|---|---|---|---|---|
| *Name* | *1* | *10* | *20* | *30* |
| s1 | 692.71 | 123.49 | 98.63 | 77.88 |
| s2 | 402.31 | 74.53 | 60.54 | 48.52 |
| s3 | 64.64 | 12.58 | 10.16 | 8.00 |
| s4 | 117.12 | 22.50 | 18.53 | 14.40 |
| s5 | 607.55 | 104.76 | 88.55 | 74.04 |

# Single vs Multi Cut

Table: Comparison of single vs multi cut L-shaped method

| | *Multi-cut* | | | *Single-cut* | | |
|---|---|---|---|---|---|---|
| *Name* | *Time* | *Gap* | *Iter* | *Time* | *Gap* | *Iter* |
| s1 | 686.81 | 0.4 | 30 | 708.91 | 24.28 | 30 |
| s2 | 406.39 | 2.51 | 30 | 455.63 | 33.08 | 30 |
| s3 | 58.16 | 0 | 4.8 | 214.2 | 0 | 19.6 |
| s4 | 105.99 | 0.01 | 14 | 223.77 | 11.88 | 30 |
| s5 | 579.02 | 0.03 | 14.4 | 1172.27 | 12.36 | 30 |

# Column Caching

Table: Time comparison for caching columns between iterations

| *Instance* | *Caching* | *No caching* |
|---|---|---|
| s1 | 686.78 | 715.77 |
| s2 | 399.28 | 422.96 |
| s3 | 62.31 | 61.52 |
| s4 | 112.8 | 105.6 |
| s5 | 615.87 | 585.76 |

# Table of Contents

# Summary

- Two-stage stochastic programming model to manage uncertain delays.
- Differentiates (planning) rescheduling and (day-of-ops) delay costs.
- Significant delay improvements over wide range of data.
- Recommended parameters to improve run-times.

# References


Ann Brody Guy. *Flight delays cost more than just time*. 2010.

Bureau of Transportation Statistics. *On Time Performance - Flight Delays at a Glance*. 2019.

Michelle Dunbar, Gary Froyland, and Cheng-Lung Wu. "An integrated scenario-based approach for robust aircraft routing, crew pairing and re-timing". In: *Computers & Operations Research* 45 (2014), pp. 68–86.

IATA. *Annual Review*. 2019.

Shan Lan, John-Paul Clarke, and Cynthia Barnhart. "Planning for robust airline operations: Optimizing aircraft routings and flight departure times to minimize passenger disruptions". In: *Transportation science* 40.1 (2006), pp. 15–28.

Chiwei Yan and Jerry Kung. "Robust aircraft routing". In: *Transportation Science* 52.1 (2016), pp. 118–133.